الجامعة الأردنية	442 جبر حديث 2: الامتحان الأول	الأحد 2014/7/6
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[1] Show that a finite integral domain is a field.

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[2] Let $R = M_2(\mathbb{Z}_6)$.

1 (a) Find $Char(R)$ $= 6$

(b) Let $A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$. Find A^{-1} $\quad det(A) = -1 = 5$

1 $A^{-1} = 5^{-1}\begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix} = 5\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$

1 (c) Find a non-zero element in $Z(R)$. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

[3] Show that $N(R_1 \oplus R_2) = N(R_1) \oplus N(R_2)$

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[4] Let $R = \mathbb{Z}_6 \oplus \mathbb{Z}_8$. Find the following:

1 (a) $U(R) = U(\mathbb{Z}_6) \oplus U(\mathbb{Z}_8) = \{1, 5\} \oplus \{1, 3, 5, 7\}$

1 (b) $Z(R) = (Z(\mathbb{Z}_6) \oplus \mathbb{Z}_8) \cup (\mathbb{Z}_6 \oplus Z(\mathbb{Z}_8))$

$= (\{0, 2, 3, 4\} \oplus \mathbb{Z}_8) \cup \mathbb{Z}_6 \oplus \{0, 2, 4, 6\}$

1 (c) $E(R) = E(\mathbb{Z}_6) \oplus E(\mathbb{Z}_8) = \{0, 1, 3, 4\} \oplus \{0, 1\}$

1 (d) $N(R) = N(\mathbb{Z}_6) \oplus N(\mathbb{Z}_8) = \{\bar{0}\} \oplus \{0, 2, 4, 6\}$

[5] Let R be a ring, $a \in R$, and let $Ann(a) = \{r \in R : ar = 0\}$. Show that $Ann(a)$ is a subring of R.

Let $r_1, r_2 \in Ann(a)$. Then $ar_1 = 0 \wedge ar_2 = 0$

$a(r_1 - r_2) = ar_1 - ar_2 = 0 - 0 = 0$

$r_1 - r_2 \in Ann(a)$ --- (1)

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$a(r_1 r_2) = (ar_1) r_2 = 0 r_2 = 0$

$r_1 r_2 \in Ann(a)$ --- (2)

From (1) & (2) $Ann(a)$ is a subring of R.

[6] Show that if p is a prime integer, then $\mathbb{Z}_p$ is an integral domain

$\mathbb{Z}_p$ is commutative ring with unity $\bar{1}$.

Let $\bar{a}, \bar{b} \in \mathbb{Z}_p$ such that $\bar{a}\bar{b} = \bar{0}$

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Then $ab = pk$ $k \in \mathbb{Z}$

$p | ab$ implies $p | a$ or $p | b$ since p is prime.

Hence $a = pk_1$ or $b = pk_2$ $k_1, k_2 \in \mathbb{Z}$

i.e. $\bar{a} = \bar{0}$ or $\bar{b} = \bar{0}$

Therefore $\mathbb{Z}_p$ is an integral domain.